RADIANT AND MOLECULAR TRANSFER IN A PLANE SYSTEM WITH SHADOW

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An algorithm is proposed for the numerical solution of the integral equation for the transfer of a radiant or molecular flux in a two-dimensional channel of arbitrary configuration. Results of calculations of molecular transfer by the proposed method and the Monte Carlo method are presented.

The steady-state problem of transfer of a radiant or molecular flux (these problems are equivalent here) in a two-dimensional (plane) channel of arbitrary configuration is considered, and an algorithm is proposed for the solution of the integral transfer equation for the effective flux density. The distinguishing feature of this algorithm is that it enables the region of visibility (the portion of the channel surface which is visible from an arbitrary point M lying on it) to be determined for a channel of complex geometry. Existing algorithms (see, for example, [1, 2]) require for their use the preliminary assignment by some means of the region of visibility, which is inconvenient and time-consuming. The proposed algorithm can be used to solve optimization problems, for example, the problem formulated in [3]. The physical properties of the surface may be arbitrary functions of position.

The transfer of a radiant (molecular) flux subject to diffuse reflection and transmission in a closed system formed by a contour L (Fig. 1) is described by a linear integral equation of the second kind for an effective radiant (molecular) flux density z(M),

$$z(M) - \gamma(M) \int_L z(N) K(M, N)\, dl_N = f(M), \tag{1}$$

where the points M and N lie on the contour L; λ(M) and f(M) are, respectively, the reflection coefficient and the flux density of the characteristic radiation (gas evolution) at the surface (and are given, arbitrary functions of the point M); K(M, N) is a Fredholm kernel depending on the law of interaction of the particles with the surface and satisfies the condition $\int_L \int_L K^2(M, N) dl_M dl_N < \infty$; dl_M and dl_N are elementary areas at the points M and N, respectively. The kernel K(M, N) differs from zero only for pairs of points M and N lying on a single straight line that does not intersect the contour between these points, i.e., only when elements of the contour L do not cast a shadow upon one another. The main difficulty in solving Eq. (1) is to determine the region of visibility, inside which $K(M, N) \neq 0$.

One of the simplest and most convenient methods of solving Eq. (1) is the zone method, in which the arbitrary functions z(M), f(M), and λ(M) are approximated by step functions z*(m), f*(m), and λ*(m), where m is the zone number. By the usual means (see, for example, [4, 5]), we write a system of linear algebraic equations of order μ

$$z^*(m) - \gamma^*(m) \sum_{n=1}^{\mu} z^*(n) \varphi(m, n) = f^*(m),$$
$$m = 1, 2, \ldots, \mu, \tag{2}$$

where μ is the number of zones;

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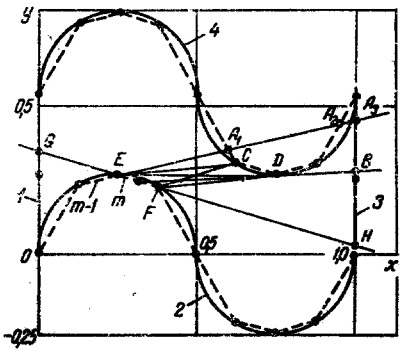

Fig. 1. Cryopump cell No. 1: 1) inlet; 2, 4) walls; 3) cryosurface.

$$\varphi(m,\ n) = \frac{1}{l_m}\int_{L_m}\int_{L_n} K(M,\ N)\, dl_N dl_M \tag{3}$$

is the mean angular coefficient; l_m is the contour length in the m-th zone; L_m, L_n are the portions of the contour L belonging to the m-th and n-th zones, respectively. In order to solve the system in Eq. (2), it is necessary to calculate the matrix of coefficients $\varphi(m, n)$, which involves finding the region of visibility. Solving the system of linear algebraic equations in Eq. (2) gives an approximate solution of Eq. (1). The accuracy of the solution rises with decrease in the dimensions of the largest zone.

The algorithm of the proposed method is as follows.

1. Divide the contour L into μ zones with coordinates x_m, y_m ($m = 1, 2, \ldots, \mu$) at the vertices. Within the limits of each zone, approximate the contour L by the straight line $y = a_m x + b_m$, where $a_m = (y_m - y_{m-1})/(x_m - x_{m-1})$ and $b_m = y_m - a_m x_m$, i.e., construct the Loman contour L*. For the sake of being definite, number the zones counterclockwise.

2. Find the numbers of the zone boundaries (the furthest is simply the boundary) g_{m1} and g_{m2} with which the m-th straight line $y = a_m x + b_m$ intersects. If there are several such points of intersection [for example, points A_1, A_2, and A_3 for the $(m-1)$-th zone in Fig. 1], choose the one closest to the m-th zone [A_1 for the $(m-1)$-th zone in Fig. 1]. The boundaries are determined in such a way that $g_{m1} \leq i$ but $g_{m2} \geq i$, where i is the number of a zone visible from the m-th zone. Store these boundary-point coordinates u_{m1}, u_{m2} (for example, the points G and H for the m-th zone in Fig. 1).

3. Find the vertex of the zone of contour L* that casts a shadow on the m-th zone (vertex D in Fig. 1) by comparing the sine of the angle of slope ω_s of the straight line passing through the center of the m-th zone and the vertices with numbers $s-1, s, s+1$ ($s = 1, 2, \ldots, \mu$). If

$$(\sin\omega_{s-1} - \sin\omega_s)(\sin\omega_{s+1} - \sin\omega_s) > 0, \tag{4}$$

then the s-th vertex casts a shadow on the m-th zone.

4. Determine the point at which the straight line passing through the center of the m-th zone and the s-th vertex (point B in Fig. 1) and store it. The zone containing this point and the s-th zone are the boundaries g_{m3k} and g_{m4k} of the region casting a shadow on the s-th vertex. The boundaries are determined in such a way that $g_{m3k} \geq i$ but $g_{m4k} \leq i$, where i is the number of a visible zone; k is the number of the shadowed region ($k=1, 2, \ldots, \varkappa$).

5. Comparing the numbers of the zones with boundaries g_{m1}, g_{m2}, g_{m3k}, g_{m4k}, investigate whether they lie in the visible region. The algorithm allows $\varkappa$ shadowed regions to be taken into account.

6. Calculate the matrix of coefficients for Eq. (2). If the law governing the interaction of the particles with the surface is cosinusoidal, the mean angular coefficient is

$$\varphi(m,\ n) = \frac{1}{l_m^*}[(l_{ED} + l_{FC}) - (l_{EC} + l_{FD})], \tag{5}$$

where l_m^* is the length of the m-th zone; l_{ED}, l_{FC}, l_{EC}, l_{FD} are the lengths of the segments joining the points E and D, F and C, E and C, and F and D, respectively (Fig. 1). If m or n coincides with one of the boundaries, then to calculate $\varphi(m, n)$ use the points u_{m1}, u_{m2}, u_{m3k} obtained earlier and also Eq. (5).

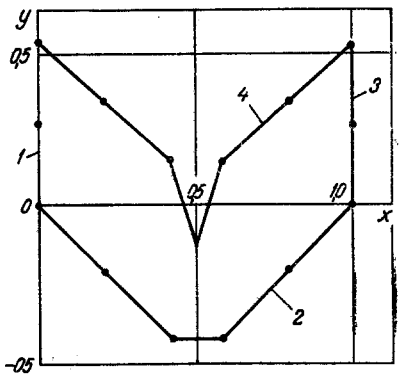

Fig. 2

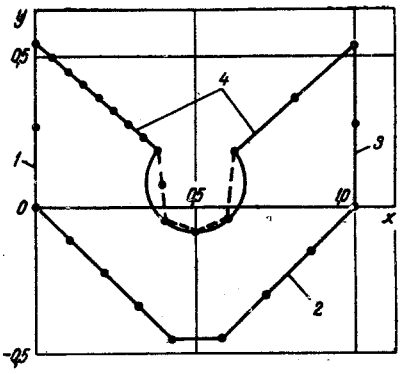

Fig. 3

Fig. 2. Cryopump cell No. 2: 1) inlet; 2, 4) walls; 3) cryosurface.

Fig. 3. Cryopump cell No. 3: 1) inlet; 2, 4) walls; 3) cryosurface.

7. Solve the system of linear algebraic equations in Eq. (2).

Using this algorithm, a program was written for the solution of the integral equation for the transfer of radiant or molecular fluxes in the considered class of systems with a cosinusoidal law governing the reflection and transmission of particles. The program was written in the language Algol-60 for a TA-IM translator. It provides for the division of large zones into smaller subzones, which allows Eq. (1) to be solved with sufficient accuracy for contours with large straight-line sections. If an M-222 MOZU computer is used, the maximum number of zones may reach 30. For $\mu = 20$ zones, the time of solution is ~ 10 sec (disregarding the time spent in translating the program).

As an example of the use of this program, the molecular-capture coefficient $\alpha = Q_3/Q_1$ of one cell of a cryocondensation pump was calculated (Figs. 1-3). Here, Q_3 is the resultant flux at the cryosurface 3 with condensation coefficient $1 - \gamma_3 = 1$; Q_1 is the diffuse flux entering the cell through the plane inlet 1 with condensation coefficient $1 - \gamma_1 = 1$. On surfaces 2 and 4, let $\gamma_2 = \gamma_4 = 1$. The capture coefficient was calculated using the Monte Carlo method (α_1) [6] and by the method proposed above (α_2) for the cells shown in Figs. 1-3. The results are shown in Table 1. Here δ is the relative error of the Monte Carlo method for a confidence level of 0.95; μ is the number of zones into which the contour L is divided (the division is indicated in Figs. 1-3 by dots; the initial contour L is shown as a continuous line, and the approximating contour L* as a dashed line). These results indicate that, for the contour formed by Loman lines (Fig. 2), the given method gives practically the same results as the Monte Carlo method. For curvilinear contours the result depends on the accuracy of approximation of the contour L. The less contour L* deviates from contour L, the more accurate the result. In comparison with the Monte Carlo method, the proposed method has the advantage that it requires considerably less machine time (by a factor of approximately 60 for $\mu = 25$, $\delta = 0.05$, and $\alpha \approx 0.25$), which allows it to be used for the solution of optimization problems. In addition, the initial information is introduced in simpler form and requires less time for its preparation.

Thus for the solution of the problem of the transfer of a molecular or radiant flux in a two-dimensional

TABLE 1. Molecular-Capture Coefficients of Cryopump Cells

Fig.	α_1	δ	α_2	μ	$\Delta = \frac{\lvert\alpha_1 - \alpha_2\rvert}{\alpha_1}$
1	0,197	0,067	0,218	20	0,107
2	0,216	0,060	0,212	21	0,0185
3	0,165	0,050	0,180	27	0,091

cavity or channel, the given method is preferable to the Monte Carlo method.

NOTATION

L, initial contour; L^*, contour approximating contour L; z(M), f(M), γ(M), arbitrary functions of the point M; z*(m), f*(m), γ*(m), step functions approximating the functions z(M), f(M), γ(M); μ, number of zones; φ(m, n), mean angular coefficient; l, contour length; α, molecular-capture coefficient; δ, relative error.

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HEAT TRANSFER OF THERMISTORS IN A NONUNIFORM ELECTRIC FIELD

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The results of an investigation of the action of dc and ac electric fields on the heat transfer of thermistors are described.

In recent years both in the Soviet Union and abroad increased attention has been given to finding new methods of improving heat transfer based on the use of electric fields. The basic principle of this method is the fact that under the action of intense electric forces in liquids and gases additional disturbances arise which under certain conditions can be localized in a narrow region of the boundary layer which has the highest thermal resistance and is therefore essentially a controllable heat transfer. The electroconvection disturbances that arise lead to a considerable increase in heat transfer.

However, despite the promising possibilities of the new method it has not been investigated to any great extent either theoretically or in practice. The theoretical assumptions and experimental results of different investigators are often questionable and even contradictory. This relates, first of all, to the nature of the action on the heat transfer of dc, ac, and mixed electric fields, and also to the quantitative estimates of the increase in heat transfer as a function of the field strength, the temperature difference, the configuration, the diameter of the heat-transfer and high-voltage electrodes, their mutual position, the temperature of the surrounding medium, etc. [6]. The methods for the experimental investigation of the effect are also far from ideal. Thus, in all the publications known to us [1-7] the heat transfer of the heated conductor has been investigated. The high-voltage was applied either to a coaxially situated conducting cylinder or to a plate placed parallel to the conductor. Hence, in all these cases the heat transfer of a conductor in a nonuniform electric field was

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